The Back of Beyond

Elie Ayache takes a step beyond essential uncertainty to tie up a few loose ends

By 'stochastic process' we mean the representation, *in writing*, of events unfolding in the unknown future, a writing made possible by means of a stochastic differential equation together with a state space *ascribed in advance*. Also we mean this whole tendency, never to forego writing and to always react, whenever it turns out that the stochastic process is not adapted to reality, by saying that the right process should have been *written*, from the start, under more general conditions. By 'stochastic process' we in fact mean the tendency never to face actuality and to always regress towards the ways that "things should have been written ideally"; we could have called it 'stochastic regress' instead. In fact, this is the tendency tending towards the "ideal end of writing." It believes that the ultimate stochastic process, capturing the random changes of the underlying as well as the changes of the parameters – even of the whole type – of the random generator, will be written one day, then handed over to the ultimate calibration attempt. In sum, a s*tochastic process* is to our mind the probabilized counterpart of the temporal B-series. Recall that the B-series is that representation of time which is totally insensitive to the singularity of the *now*. B-theoretic events are ordered relative to each other, in terms of *before* and *after*, and made available to the mathematically formulable conceptions of time – those typically involving theoretical representation – as

one immutable geometric series.

Now if you think that unpredictability is of the essence of time and that the future is what is essentially unknown, you may start thinking we should appeal, on the contrary, to the A-theoretic conception of the time series whenever the *content* of the successive events becomes of interest and we are no longer satisfied with the process of ordering them formally – ordering them as if they were nameless variables. If you believe, in other words, that *there is no time*, really, outside the internal clock of a stochastic process, then it will seem to you that the A-series, and the special place it keeps for the *now*

and for *actuality*, are alone capable of accounting for the interplay of probability and realization, of potentiality and actuality, that constitutes the essence of the stochastic process. We tend to disagree. We like to strike the difference between the B-series and the A-series, not just in the terms outlined by Yourgrau, but more generally, in terms of the difference between what is representable and what is not, between what can be made present to the mind or readily prepared for the survey of the mind and what can only be performed in actuality. This is the difference between representation and performativity, between what can be written in advance and

cast in an envelope, and what requires on the contrary that the *actual performance* of writing remain an indispensable part of the open-ended process of writing (or, shall we say, of the *progress* of writing).

What we are really saying is that there are two notions of actuality involved here, a weaker notion and a stronger notion. Surely enough, every stochastic process does include a notion of actuality, but this is simply derivative on the notion of probability that lies at its foundation. Technically, it translates into the notion of filtration and adapted process. (It says: innovation in a stochastic process, when the latter is adapted to the filtration, unfolds at the same time as the filtration branches out). The stochastic process has this weaker notion of actuality *inscribed* in it. Actuality, we may say, has here been defeated by the writing process. It has been written *down*. This is the purpose of the stochastic process. The stronger notion of actuality, however, is that actuality which determines what stochastic process we should be using in the first place, better still, whether we should be using a stochastic process at all. Think of it as the *actuality of trading* as opposed to theoretical representation: your best trader against your best quant.

The analogy with Quantum Mechanics will help us again. Recall that Quantum Mechanics, according to Bitbol, is but "a form of meta-contextual probability theory." While the classical probability functions, as formalized by Kolmogorov, assign a number between 0 and 1 to "events" defined as subsets of elementary events, and while this whole collection of subsets, including the empty set and the total set, obeys the classical laws of reunion and intersection given by Boolean algebra, the quantum probability functions generalize the picture to cases where the context of experiment can no longer be bracketed and kept implicit, and generalize the Boolean algebra to a structure known as 'ortho-algebra.' Ortho-algebras are richer structures than Boolean algebras. They contain the Boolean algebras as sub-structures. To each different experimental context, also known as 'quantum measurement,' there corresponds a range of possible outcomes and a particular Boolean algebra overlain by the classical Kolmogorov probability func-

tion. When the contexts are compatible with each other and the particular order that they are brought into play through the total experiment does not matter, the corresponding sub-algebras can merge, the sets of possible outcomes can simply add up, and the classical probability function simply have its domain of definition extended. The reference to a context, or to different contexts, really goes unnoticed in this case.

However, when the contexts cannot be conjoined (typically, when dealing with canonically conjugate variables such as quantum position and momentum), and each has to remain distinguished by its particular range of outcomes, the "object" we are dealing with has to be represented at a higher level of generality and abstraction than in the usual cases. It can no longer be defined as "this, which can have this property or that property, produce this outcome or that other outcome, with this given probability." It has to be defined at the level where we contemplate a range of possible alternative contexts, not just a range of possible outcomes. Indeed our "object" can behave, now like particle, now like wave, depending on whether the context of experimental disclosure is suited for a particle or for a wave. Now it is the need that we nevertheless *articulate*, relative to one another, the different contexts and the different sub-logics that we can no longer simply blend, and the need to represent our object with one symbol, which lead us to the formalism of the wave function and to Quantum Mechanics as meta-contextual, or generalized, probability theory1. And now you can see that there are two notions of actuality involved in Quantum Mechanics as well: the actuality of the particular possible outcome, and the actuality of the context. While the first is the business of chance and probability in the ordi-

nary sense, the second cannot be left to "take care of itself." It largely depends on the pragmatics of the experimentation, that is, on the actual presence of an experimenter and on her actual decision to prepare this or that experiment, and to actualize this or that context.

An epistemological occasion
At this point, Taleb would jump in and object: "But it ain't physics!" We couldn't agree more. It hasn't been physics for some time now. It is all about epistemology! Isn't that Taleb's favorite domain? Quantum Mechanics is not "just another physical theory." It is a critique of the theory of knowledge and the theory of probability. Surely the reader must have guessed that we are looking for a generalization of probability theory, suitable for quantitative finance and its epistemology, the methodology of risk, along the lines of Quantum Mechanics. Or rather, we are looking to *redefine quantitative finance as being just this generalization!* Stochastic processes are definitely not the whole story. They act only in particular contexts, and miss entirely the higher, meta-contextual, picture. And quantitative finance, this enlarged science we are seeking, is definitely not a branch of stochastic calculus or econometrics. "Generalizing" the stochastic process – the quant's article of faith! - should not proceed by way of generalizing its *form*, by multiplying the number of its parameters, or by formalizing the random changes of the random generator. For this would be mistaking the meaning of 'generalization' brought up by Quantum Mechanics. Quants, the likes of Lipton, Hagan, Carr and Duffie, may have been optimistic about this prospect. Taleb is plain pessimistic. They say risk is recursively representable, he says essential uncertainty is irreducible to risk. But neither

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To repeat, the opportunity that is offered us by quantitative finance is plain epistemological. Don't expect from it any "hard" consequence (in the sense of the "hard sciences"). Don't expect our philosophical critique to land the "wave function" of the market the same way that the wave function of the whole physical universe is in theory conceivable. True, when we say that every stochastic process that has been proposed to date to model the behavior of the underlying market, and consequently to price derivatives and to manage risk, is in fact confined to a particular context and to a particular choice of relevant risk measures (for instance, the context where the volatility of the underlying stock, the volatility of interest rates – to name the quantitative variables – may matter, where even the country's political stability may matter, but where the temperature of the air will not matter), when we say that every such proposition misses the higher point of view where a meta-contextual "object" can be articulated at last and encapsulated in a symbolism, true we may sound as if we were aiming at such an advance, and were about to propose a formalism for quantitative finance akin to the quantum Hilbert spaces and to the quantum ortho-algebras. Unfortunately, such a feat is not in sight, and such has not been our purpose. Again, it is the *critical philosophical* analogy that

we seek in Quantum Mechanics, not the physical, mathematically formulable, example. Our whole point is to highlight the meta-contextual perspective without necessarily casting it into mathematical symbols, and to replace Taleb's 'essential uncertainty' with our 'essential actuality.' Recall that the inseparability of the quantum phenomenon from the context of its manifestation is what brought us the stronger notion of actuality. Is Taleb aware of a similar inseparability in quantitative finance? Is he aware that the complete picture of risk does not just stop at a particular context and a particular model for risk? Of course he is. Is not his 'central problem of risk management' indeed directed towards the limitation of the classical, intra-contextual, "Kolmogorovian" view, and almost longing for our inter-contextual, meta-epistemological perspective? Only Taleb does not attempt to *articulate* this from above. He prefers radical skepticism and retreat into silence. His 'knowledge' stops at old knowledge. When the optimist about old knowledge persists in generalizing the stochastic process, and in framing the random changes of the random generator, Taleb's pessimism, not leaving the plane either, answers him with the end of that knowledge and with essential uncertainty. Both their epistemologies remain centered around the *knowledge of* the underlying probability distribution and the belief that it can be found. The only difference is that the optimist assigns a positive sign to that belief and the pessimist a negative sign. But they both display the same structure, that of 'a knowing subject *facing* a world.'

Essential actuality

By contrast, the new knowledge and the new epistemology we are seeking do not look at the world (or the market) as a domain of investigation separate from the investigator. Neither do we seek "to examine the market from close to" and try to pin down its random generator, nor do we wish to repel from the market and retreat into an attic. (As if we were free to perform such advancing and retreating actions!) The knowledge we are interested in is not the *knowledge of* the ultimate random generator, nor is it the knowledge of the inaccessibility of that kind of

knowledge. In our sense, the real epistemological lesson, if any, to be learnt from quantitative finance, should lie at the same level as the lesson from Quantum Mechanics. It should tell us what there is to be known about our own situation of market participants who *have to* design quantitative tools and use them, when critical reflection about the theory and the attached conditions of knowledge can lead to utter skepticism like Taleb's. We wish to step beyond *essential uncertainty*, and see what further epistemology can be had. We called it 'essential actuality.'

In a nutshell, we speak of 'essential uncertainty' when no probability can be meaningfully assigned to possible future results, or in other words, when no one probability model can be preferred to another. But we speak of 'essential actuality' when the whole theoretical and representational framework is no longer suitable, and something else – actuality – has to enter in the composition of our epistemology. It is one thing to argue that no particular theoretical model can work (then to step back into skepticism); it is another to realize that the whole inclination to theorize and to represent the world by means of theoretical scripture is no longer appropriate as epistemology (and consequently to move forward towards a new epistemology). Recall that actuality is, in our own words, that "solid block precisely facing, and almost contradicting, theoretical representation." And think that theoretical representation is definitely not sufficient to account, say, for quantum reality. You can try and write the most complete and the most global wave function that can be, the wave function of the whole physical universe if you wish. Schrödinger's equation will then in theory describe for you the evolution of that wave function. But unless you specify, in a *completely extra-theoretical way*, that is, in actuality (that is, as of *now*, *as of here*, and as of *you*, the actual experimenter), what particular context you are immersed in, and what particular measurement you are after, the state of the world will continue to unfold as an undifferentiated superposition of states, and the description of reality will be incomplete[2].

Remember the two senses in which Quantum Mechanics is the most general theory of knowledge. It is most general in the sense that every single bit of the physical universe;

every single object and every single measuring device can be made part of the wave function. And it is most general in the sense that this "semantic completeness," this capacity to embed the universe in one single scripture that *no one in particular has yet been picked up to read*, will in effect translate into the meta-contextual theory of predictions we've been talking about, when the notion, not yet the particular instance, of the "situated reader" or the "situated experimenter" is brought into play, and will result in the *generalized* probability theory with the strange probability interference term and the necessity to actualize the context, when the quantum measurement process is instantiated as one particular measurement interaction following one particular quantum preparation. The lesson to take away from Quantum

Mechanics is that utter generality and completeness, in the very direction dreamt of by every theoretician and every "theoretical writer," the direction leading to the "ideal end of writing," admits as correlate a generality in the perfectly orthogonal direction: the necessity to realize that properties are not inherently borne by objects and that, *in general*, an explicit reference to the actual context of generation of the observed phenomenon is required. The more you generalize in the way of theoretical representation, the more generally the need is felt for the one thing escaping theoretical representation: actuality. This is a very simple lesson.

Another epistemological occasion
Now to go back to quantitative finance, it is clear that the general quantitative trend has been to

follow the road of theoretical representation, and to pursue the dream of the ultimate stochastic process and the ultimate management of risk. Taleb's *essential uncertainty* (or central problem of risk management) marks the end of that road, and much sooner that you had thought ("What can go wrong will go wrong," "What by definition can hurt you is what you expect the least.") However, both attitudes rely on the (metaphysical) assumption that there *exists* something like the probability distribution of the market. Markets being essentially unpredictable, it is only natural that probability and stochastic processes should come to mind as the form of science suited for the object domain. And now Taleb's radical skepticism strikes us as an epistemological overlay. His essential uncertainty acts as a roadblock against any knowledge claim we

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may make *about* the probability distribution. No past data can help us infer the parameters of the random generator. We may not even be in a position to assume that the random generator is of a certain general type. For all we know, the random generator may itself be randomly changing, etc., etc. There must be something, having to do with the complex nature of markets, which accounts for this opacity of knowledge. Perhaps feedback and self-fulfilling prophecies, or other highly non linear phenomena such as speculative bubbles, should be invoked to explain why the "uncertainty principle" always has the last word in the markets and why, no matter what you do, chances are your framework of risk will be overwhelmed by risk. You can easily imagine what complex theories – and what theories of complexity –can develop from here. All these

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elaborations strike us, however, as being based on the single assumption that the market *is out there* as one big strange generator of outcomes facing us, and that there is something to know, or something we will never know, about it. A single unshakable metaphysical assumption leads to a high-rise edifice of ever more complex stochastic representations and their corresponding epistemologies, from advanced derivative pricing technology to chaos theory to neural networks, and to the corresponding skepticisms.

Our bet is that once the metaphysical assumption is shaken and removed, we will end up instead with a very simple epistemology. We will dispense, once and for all, with exploring the endless intricacies of this mysterious entity known as the market, and we will derive our new epistemology from first principles. Perhaps a reflection on the prior meanings of prediction and actuality, on the meanings of theoretical representation and actual fact, the meanings of anticipation and action, the meanings of context and meta-contextual formalism, indeed on the whole meaning of probability, can do a better job at solving – or rather dissolving – a vexed problem than a wealth of theories and counter-theories, exactly like it did in Quantum Mechanics. The same way that the elementary particles really furnished us with an *epistemological occasion*, quite independently of their special ontological status of "being an elementary particle" - or perhaps this limiting test for our epistemology was due to their elementary nature after all, but then from our epistemological-structural point of view, we can again invert the existential statement and instead of saying: "There is an elementary particle and it commands this peculiar epistemology" we can say: "There must exist such a limiting case in our general epistemological scheme, if only for the sake of meta-epistemological reflection, and it so happens that that case is borne out by the elementary particles" - perhaps the market can furnish us, in like manner, with another sort of extreme epistemological occasion, quite independently of its wholly complex nature.

Old knowledge vs. new knowledge

And what is this epistemological occasion? Taleb's 'essential uncertainty' is the stultifying statement that the next market move, the next tick, may have nothing to do with any *given* probability model. This forces you into essential resignation, or into a quest of the wildest sort: the quest for a generator model which is "not yet given yet must have been ideally given from the start." (This paradoxical sounding formulation is due both to the tension between essential uncertainty, which says: "No *given* model is a model, this is the end of the road," and to the incapacity, for all that, to leave the plane of theoretical representation and its epistemology, which curls back to the beginning of writing and says: "Now that we *know*, perhaps we stand a better chance of knowing, from the beginning, what knowledge cannot know, and we can recuperate it.") Our 'essential actuality,' on the other hand, is the much simpler remark that the next tick may have nothing to do with theory as a whole, without it being suggested that your *given* theoretical tools have to be necessarily replaced. Theory is theory and there exist better theories just as there exist better tools, but you can only face actuality with actuality. Markets are essentially unpredictable, and we are precisely talking about probability and uncertainty – we certainly do not wish to trivialize this talk! - but imagine a new epistemology where probability is so to speak eliminated, and where actuality is the only thing that counts. The obvious question will be:

How would you model this? How would you model a "process of actualities"? Wouldn't we be back to the pre-theoretical, perfectly empirical, philosophy, where the market "facts" are all there is: an unconnected series that no model or theory can have a grip on, and where we "should focus only on what we do not know"?

Not quite. Our strong and enriched sense of actuality is not the same as the notion of isolated and naked Humean fact that Taleb wants us to fall back to. By contrast to Taleb, we want a job for the science and for the philosophy of the science. Our strong sense of actuality is the result of *the whole reinterpretation and the whole rethinking of probability* that quantitative finance leads us to. Neither is our 'actuality' the actualization of a probabilistic outcome (as presupposed by the Kolmogorovian probability theory and the normal view of stochastic processes), nor is it the actuality of pre-theoretical empirical reality. Our 'actuality' is the actuality we get at the end, that is, at *the opening*, of treatises like *Dynamic Hedging* and *Know Your Weapon*. When the trader steps out of such training programs, knowing his weapon the way Haug wishes him to, and understanding dynamic hedging the way Taleb has explained, will a notion like essential uncertainty really matter to him? He may be aware that his weapon is cast in a certain metal and what inner mechanism triggers the firing of the weapon (in other words, he may be aware of the whole representational framework and its shortcomings), but this is not exactly the *knowledge* that we claim he must be equipped with at the end of a successful teaching. Having digested a lot of option pricing theory and having known all there is to know about the interrelations of the various greeks, the real knowledge he now masters has nothing to do with the "past" and what has been written before, and everything to do with the future, that is, what will happen next, what will happen *now*. The trader's *new knowledge* knows nothing of what went before. When we say that the trader is inscribed between a 'before' and a 'next,' between the exhaustion of a certain theoretical framework and the "beginning" of actuality, we do not mean that representation can recuperate him too, and that he will become part of a larger "process" aligning a 'before' and an 'after' in the sense of a B-series.

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The trader himself marks the end of representation (this is why he is requirement in our new epistemology). He faces actuality free of any remnant of old knowledge. This is why essential uncertainty cannot affect him. Yet it cannot be said that he "focuses only on what he does not know" and knows only empirical, unstructured, undifferentiated, reality. For he knows his weapon and has learnt option language.

How can essential uncertainty affect someone who has used old knowledge and the representational framework for the sole purpose of practicing his weapon and knowing it, and who is now prepared to use *new knowledge* completely outside representation, completely outside the imperative of writing and framing and fixing the random generator and guessing its parameters, that is, completely "inside actuality"? What would it matter if the random generator might not have been of the type

presupposed... by the weapon? It is not the weapon that *knows* after all. It is the pair composed of the trader and his knowledge of the weapon; a pair that is now equivalent to *new knowledge*. And this new knowledge is ready for about anything, in actuality. Given the way in which new knowledge exceeds old knowledge, this excess and openness in fact relaxes the epistemological loop previously produced by essential uncertainty. That the next tick may be what it is (in actuality), yet may not be what it should be (in theory) because no given probability model could be found to apply, and the only model left is the shadowy not-yet-given-yet-must-have-been-given-from-the-start probability model, all that, all that hesitation and oscillation, is dealt with in one stroke once old knowledge and the attached essential uncertainty are left behind, and new knowledge opens up to its new designated field: essential actuality.

ENDNOTES

1 Bitbol writes: "When we try to build a meta-contextual probability formalism, with the only constraint that the axioms of Kolmogorov be verified in isolation for each separate range of possibilities, and that we use a unique symbol for the initial quantum preparation which would generate the probability sub-functions associated with each different context, we end up with a class of structures which admits the formalism of Hilbert spaces and quantum mechanical wave functions as a particular case."

2 This retraces an argument, already given by Jean-Louis Destouches (1951), according to which the "theoretically complete" representations of the world, which are supposed to be independent of the subject and which Destouches calls the "objectivist descriptions," are in fact incomplete. "In fact, writes Destouches, it is the objectivist descriptions which are incomplete, because they do not account for the different behaviors of the observers and for the freedom that they can

exercise in the choice and the conduct of the measurement operations. Objectivist descriptions present themselves as general, but in fact are not, because they are relative to the particular situation of the observed system within a particular apparatus, and it is therefore impossible to generalize the determinism that they imply without falling in contradiction with the criteria of essential indeterminism." Destouches then opposes the notion of "objectivist description" to that of "subjectivist prediction." The latter acknowledges the dependence of the observable phenomenon on the context of experimentation and the unaccountability of the choice of context within general theoretical representation. The "subjectivity" that is here involved must not, of course, be confused with psychological or anthropological subjectivity. As Paulette Destouches-Février explains: "The "subjectivity" that is in question in Destouches's analysis belongs to a transcendental level, not to the domain of psychology or anthropology."

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