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Valuing convertible bonds with 20-of-30 soft call provision

Farhad Firouzi, quantitative analyst, and Elie Ayache, co-founder and chief executive officer of ITO33, present a fast and accurate method of evaluating one of the most daunting features of the convertible bond market – convertible bonds with the soft call feature of 20 out of 30 days

ITO33 is the leading provider of software and solutions to value and risk-manage convertible bonds and equity derivatives. At a time when many vendors are merely addressing issues of scope and volume, ITO33 remains convinced that firms specialising in trading convertible bonds should also demand rigorous handling of challenging valuation problems. These can have a significant impact on the theoretical value and greeks of the traded instruments.

Introduction

A convertible bond with the soft call feature of 20 out of 30 days – or a 20-of-30 soft call – is a convertible bond that may be called by the issuer only if the stock price has closed at least 20 days above a specified trigger during the last 30 trading days. Usually, the early redemption value is lower than the conversion value; therefore, the issuer forces the conversion of the convertible bond when he calls it. There is no exact method to valuing this complex hybrid security. The Monte Carlo method[1,2] is slow and not straightforward as the 20-of-30 soft call is strongly path-dependent. This article discusses and compares our valuation method with the equivalent 1-of-1 soft call approximation and the *N*-consecutive-day soft call approximation. We will also see that our valuation method is fast and accurate for all the values of the stock price and handles the closing history[3] well.

Example set-up

We consider the following convertible bond as the main example of this article, in effect, all of the examples have the following parameters unless otherwise stated:

Nominal value	100
Conversion ratio	1
Convertible from	September 1, 2011
Convertible until	September 1, 2016
Maturity	September 15, 2016
Valuation date	September 1, 2014
Coupon rate	6% annually
First coupon date	January 3, 2012
Soft call start date	September 1, 2014
Soft call trigger	120
Early redemption value	100
Closing history	None[4]

We consider a fixed *3%* annual interest rate and a log-normal diffusion for the stock price with a volatility of *20%*.

Accuracy and complexity of our valuation method

The only exact valuation[5] of an *M*-of-*N* soft call consists in using 2^N parallel partial differential equations (PDEs) with the closing history as state variable. This method has a complexity of $O(2^N)$ and becomes useless as N increases, typically as N exceeds 10. Consequently, there is no benchmark available for the 20-of-30 soft call; however, we can test our valuation method for some simple cases such as 3-of-5 or 7-of-10 soft call. As illustrated in table A and figure 1, our method is quite accurate; the maximum error in delta for the case of 7-of-10 soft call is around the spot price of *123* where the exact value and our estimation are *67.7%* and *67.1%*, respectively, which is negligible, considering the sharp change of the delta at this range.

Our valuation method is also PDE-based, however, its approximation is such that it has a linear complexity of $O(N)$ so, unlike the exact method with the exponential complexity, it doesn't have any limitations on N (table B).

A Exact value versus our estimation of 3-of-5 and 7-of-10 soft call deltas

	3-of-5		7-of-10	
S	Exact	Approximate	Exact	Approximate
90	34.4%	34.4%	35.8%	35.8%
100	33.7%	33.7%	36.4%	36.5%
110	24.5%	24.5%	27.6%	27.7%
115	21.6%	21.6%	23.9%	24.0%
120	29.7%	29.6%	27.4%	27.3%
122	72.2%	72.1%	53.9%	53.4%
123	86.2%	86.1%	67.7%	67.1%
124	93.8%	93.7%	78.9%	78.4%
126	99.0%	98.9%	92.5%	92.3%
130	100%	100%	99.4%	99.4%

[1] *Beveridge CJ and MS Joshi (2011), Monte Carlo bounds for game options including convertible bonds,* Management Science *57 (5), pp. 960–974.*

[2] *Crépey S and A Rahal (2012), Pricing convertible bonds with call protection,* Journal of Computational Finance *15 (2), pp. 37–75.*

[3] *The stock closing prices on the last 30 trading days*

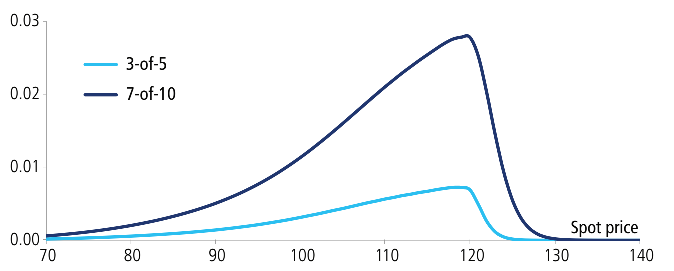
[4] *By no closing history, it is meant that the stock price has closed consecutively below the trigger on each of the last 30 trading days.*

[5] *Ignore truncation error of PDEs.*

B Computation time[6] of our method versus exact method for different soft calls

	3-of-5	7-of-10	20-of-30
Our method	0.2s	0.4s	1.1s
Exact method	0.6s	18.7s	–

1 Absolute error of our method in valuation of 3-of-5 and 7-of-10 soft call



Negative deltas

One important aspect of soft calls is the case of negative delta. When, for example, the spot price is close to the trigger and it has previously closed 19 consecutive days above the trigger, the probability that the issuer calls back the bond can increase very quickly as the spot price increases, which may cause the convertible bond value to decrease. Again, as there is no exact solution for the 20-of-30 soft call, we test our valuation method for the simple case of 7-of-10; we suppose that the stock price has previously closed for six consecutive days above the trigger and we investigate the delta for spot prices around *120*. As set out in table C, our method is also accurate for negative deltas.

C Exact value versus our estimation of 7-of-10 soft call delta around trigger when the stock price has previously closed for six consecutive days above the trigger

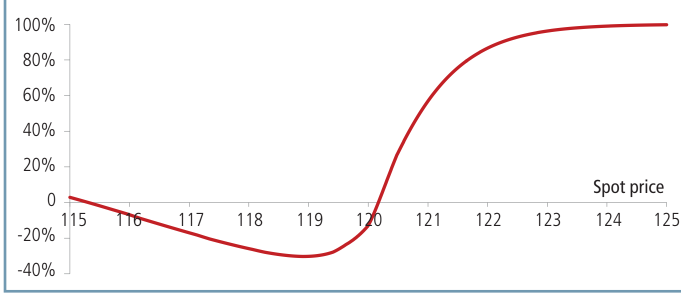
S	Exact	Approximate
115	19.3%	19.6%
116	14.1%	14.2%
117	5.5%	5.4%
118	-6.0%	-6.4%
119	-16.2%	-16.9%
120	-4.2%	-4.9%
121	59.5%	59.3%
122	87.4%	87.3%
123	96.4%	96.4%

Equivalent 1-of-1 soft call

The technique widely used in the market to value 20-of-30 soft calls is the equivalent 1-of-1 soft call. The idea is to find an equivalent trigger $B^* > B$, such that the value of the 20-of-30 soft call with trigger B is equal to the value of the 1-of-1 soft call with trigger B^*, and calculate the value of the convertible bond

[6] *Using a Core i7 processor @3.4GHz, and a 64-bit operating system.*

2 Our estimation of 20-of-30 soft call delta when the stock price has previously closed for 19 consecutive days above the trigger



through the simple case of 1-of-1. There is no exact solution for B^*, and it is approximated as follows:

1. For a given spot price, calculate the probability to activate the 20-of-30 condition using the Monte Carlo method.
2. Over the same Monte Carlo trajectories used in step 1, search for the equivalent trigger $\hat{B}^*$, which would give the same probability to activate the 1-of-1 condition.

$$Pr\left[\exists\, j \le L: \sum_{i=j-29}^{j} 1_{\{S(t_i) \ge B\}} \ge 20\right] = Pr\left[\max_{i \le L} S(t_i) \ge \hat{B}^*\right]$$

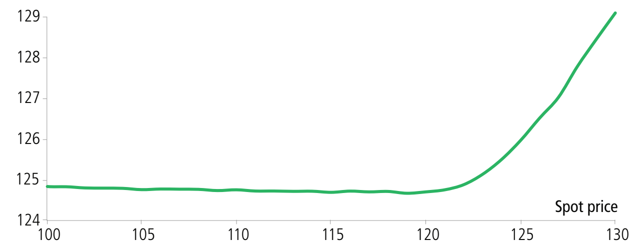
Where t_is are the observation times – closing times – and L is the length of the soft call period (two years in our example). The equivalent 1-of-1 method has some major disadvantages, as follows:

- Mathematically, we cannot prove that $\hat{B}^*$ is equal to B^*.
- $\hat{B}^*$ is time-consuming to estimate, as we need to use the path-dependent Monte Carlo method for the whole soft call period. In our example, it takes about one minute to estimate $\hat{B}^*$ over 10^4 trajectories, which is an immense amount of time compared to the computation time of around one second it takes our method to estimate the convertible bond value.
- From figure 3, we find that $\hat{B}^*$ is a function of the spot price, and generally is also a function of the volatility and the closing history, so it needs to be recalculated in real time.
- The value of the convertible bond is sensitive to the equivalent trigger, so it needs to be estimated accurately. In the case of 7-of-10, for example, at $S=115$, the exact value and the value obtained by our method are *121.05* and *121.07*, respectively. Meanwhile, the lower (upper) bound[7] of the equivalent 1-of-1 trigger, estimated over 10^4 standard Monte Carlo simulations, is *122.24 (122.40)*, giving a convertible bond value of *121.03 (121.13)*. This implies that this method, using 10^4 simulations, is less accurate than our method (see figure 4 for the absolute error of this method for other spot prices). For the case of 20-of-30, at $S=115$, the lower (upper) 1-of-1 value – corresponding to trigger *124.66 (124.92)* – is *122.46 (122.60)*; while our method gives *122.55* for the convertible bond value.
- Using this method, it is difficult to achieve a good estimation of the convertible bond delta, as the equivalent trigger is a function of the spot price:

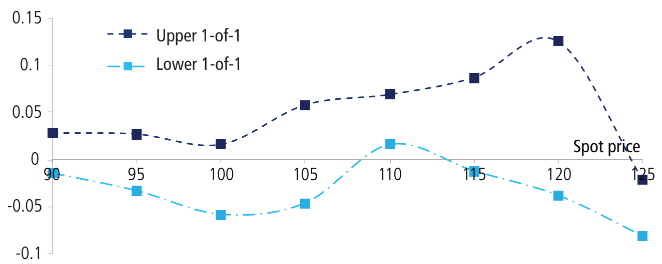
$$\frac{dV(S, B^*(S))}{dS} = \frac{\partial V}{\partial S} + \frac{\partial V}{\partial B^*}\frac{dB^*(S)}{dS}$$

[7] *Upper and lower end-points of the 95% confidence interval.*

3 Equivalent 1-of-1 trigger corresponding to 20-of-30 trigger of 120



4 Absolute error of equivalent 1-of-1 approximation of 7-of-10 soft call corresponding to lower and upper bounds of equivalent trigger estimated over *10⁴* trajectories



For the case of 7-of-10, for example, at $S=123$ the lower and upper bounds of the equivalent trigger are *122.94* and *123.19*, respectively; delta of the 1-of-1 soft call with a fixed trigger of *122.94 (123.19)* is *44.4%* (*36.0%*); while delta of the 7-of-10 soft call is *67.7%* (see table A), which implies the second term of the previous equation ($\frac{\partial V}{\partial B^*} \frac{dB^*(S)}{dS}$) cannot be ignored.

N-consecutive-day soft call

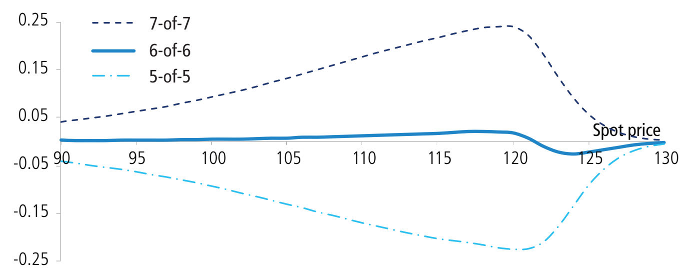
The *N*-consecutive-day (or *N*-of-*N*) soft call is another valuation method of 20-of-30 soft calls. The idea is to increase *N* the number of consecutive days, instead of the trigger, to match the value of the original soft call and solve it through the simple case of *N*-of-*N* by using $N + 1$ PDEs. Unlike the trigger, *N* is discrete so we may not be able to perfectly match the original soft call, but it is easy to find the appropriate *N* and the error is satisfactory in the absence of closing history (see figure 5). *N* can be found in a similar way to the previous section: the probability of activating the 20-of-30 condition is equal to the probability to activate the *N*-consecutive-day condition; from table D, we find that six-consecutive-day is a good approximation for the 7-of-10 soft call.

The *N*-consecutive-day approximation is not robust when the soft call period has already started and we have to take into consideration the stock price closing history. In figure 6, in the presence of two similar closing histories, we compare the 20-of-30 delta, calculated by our valuation method, with different *N*-of-*N* approximations. Vectors of closing history for these two examples are almost the same – for the first example, we have 10 consecutive days above the trigger while, for the second, we have nine consecutive days above followed by one day below the trigger. As we can see, 20-of-30 deltas have similar shapes for both examples, while they are completely different for the *N*-of-*N* soft calls, since the number of past consecutive days above the trigger is reset to 0 for the second example.

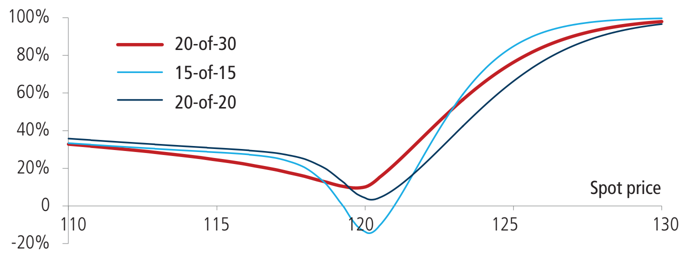
D Probability of activating soft call condition when *S=110* and *B=120*, over *10⁴* trajectories

7-of-10	5-of-5	6-of-6	7-of-7
70.4%	71.0%	70.3%	69.6%

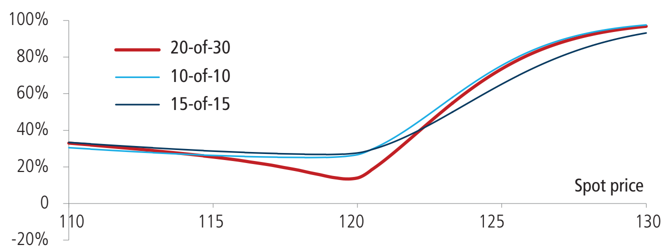
5 Absolute error of different *N*-consecutive-day approximations of 7-of-10 soft call



6a Estimation of 20-of-30 delta versus different *N*-consecutive-day deltas when stock price has closed 10 consecutive days above trigger



6b Estimation of 20-of-30 delta versus different *N*-consecutive-day deltas when stock price has closed nine consecutive days above followed by one day below trigger



Conclusion

A brief discussion on different PDE-based valuation methods of 20-of-30 soft calls revealed that none of these methods are perfect: the 1-of-1 method is sensitive to the equivalent trigger, which is not easy to estimate; and the *N*-consecutive-day method is sensitive to the closing history. We showed that our valuation method is fast and accurate, doesn't need to calculate any intermediate parameters and is more robust than the common methods used in the market.

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